Fast First-Order Methods for the Massive Robust Multicast Beamforming Problem with Interference Temperature Constraints

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**Introduction**

We consider the following robust beamforming problem with interference temperature constraints in a cognitive radio network:

\[
\begin{align*}
\max \min_{\mathbf{w} \in \mathbb{C}^N} & \quad \frac{1}{\sigma^2} ||\mathbf{h}^T \mathbf{w}||^2 \\
\text{s.t.} & \quad ||\mathbf{w}|| \leq P, \\
& \quad \mathbf{w} \in \mathbb{C}^N \text{ is the beamforming vector;}, \\
& \quad h_i \text{ is the channel between SBS and the } i\text{th SU;}, \\
& \quad a_j \text{ is the estimated channel vector between the SBS and the } j\text{th PU;}, \\
& \quad \delta_j \text{ is the estimation error bounded by } d_j, \\
& \quad P^2 \text{ is the average transmit power.}
\end{align*}
\]

The interference temperature condition is equivalent to the following deterministic version:

\[
d_j ||\mathbf{w}|| + ||a_j^T \mathbf{w}|| \leq \eta_j, \quad \text{for } j = 1, \ldots, J.
\]

Then, convert this problem into the real domain:

\[
\begin{align*}
\min_{\mathbf{w} \in \mathbb{R}^N} & \quad \max_{i=1, \ldots, M} ||\mathbf{G}_i \mathbf{w}|| \\
\text{s.t.} & \quad d_j ||\mathbf{w}|| + ||a_j^T \mathbf{w}|| \leq \eta_j, \quad \text{for } j = 1, \ldots, J, \\
& \quad ||\mathbf{w}|| \leq P.
\end{align*}
\]

**LPA-SD: Linear Programming-Assisted Subgradient Decent**

We define

- \( f_i(\mathbf{w}) = \mathbf{w}^T \mathbf{G}_i \mathbf{w}, \quad i = 1, \ldots, M, \) and \( F(\mathbf{w}) = \max_{i=1, \ldots, M} f_i(\mathbf{w}); \)
- \( \delta \)-active objective functions set: \( I_\delta(\mathbf{w}) := \{i \in \{1, \ldots, M\} : f_i(\mathbf{w}) \geq F(\mathbf{w}) - \delta \}; \)
- \( \epsilon \)-active constraints set: \( I_\epsilon(\mathbf{w}) := \{j \in \{1, \ldots, J+1\} : \gamma_j(\mathbf{w}) \leq F(\mathbf{w}) - \epsilon \eta_j \}; \)
- \( \epsilon \)-tangent space at \( \mathbf{w} \): \( \mathcal{T}_\epsilon(\mathbf{w}) := \{x \in \mathbb{R}^{2N} : \mathbf{x}^T \nabla f_i(\mathbf{w}) = 0, j \in I_\epsilon(\mathbf{w})\}; \)
- projected gradient onto \( \mathcal{T}_\epsilon(\mathbf{w}) \): \( \mathbf{p}_\epsilon(\mathbf{w}) = (I - UU^T)\nabla f_i(\mathbf{w}). \)

Then solve the following LP

\[
\begin{align*}
\max_{\mathbf{t} \in \mathcal{A}} & \quad \sum_{i \in I_\delta(\mathbf{w})} \lambda_i \mathbf{p}_\epsilon(\mathbf{w}) \\
\text{s.t.} & \quad \lambda_i \mathbf{p}_\epsilon(\mathbf{w}) \geq 0, \\
& \quad \lambda_i \in I_\delta(\mathbf{w}).
\end{align*}
\]

Find the Armijo-type stepsize,

\[
\hat{\theta} = \max\{\theta : F(\mathbf{w} - \theta \mathbf{d}) \leq F(\mathbf{w}) - \gamma \theta^2 \},
\]

where \( 0 < \theta < 1 \) and \( 0 < \gamma \leq 0.5. \) Here, \( \gamma \) is a scaling parameter to ensure the feasibility of \( \mathbf{w} := \mathbf{w} - \theta \mathbf{d}. \)

**ADMM**

Firstly, introducing the auxiliary and slackness variables to split the constraints

\[
\begin{align*}
\min_{\mathbf{w}, \mathbf{d}, \mathbf{y}, \mathbf{u}} & \quad t \\
\text{s.t.} & \quad ||\mathbf{x}_i - \mathbf{u}_i|| = \sqrt{\epsilon}, \quad i = 1, \ldots, M, \\
& \quad \mathbf{d}_j ||\mathbf{y}|| + ||\mathbf{z}_j|| + \gamma_j = \eta_j, \quad j = 1, \ldots, J, \\
& \quad \mathbf{x}_i \sim \mathcal{N}(\mathbf{w}_i, \mathbf{I}/\sqrt{\mathbb{N}}), \quad \text{for } i = 1, \ldots, M, \\
& \quad y = \mathbf{w}, \quad \mathbf{z}_j = \mathbf{A}_j \mathbf{w}, \quad j = 1, \ldots, J, \\
& \quad ||\mathbf{y}|| \leq P, \quad u \geq 0, \quad v \geq 0.
\end{align*}
\]

Write down the augmented Lagrangian function and apply the standard multi-block ADMM.

**Numerical Result**

We generate the data as follows:

- \( h_1 \sim \mathcal{CN}(0, I) \) follows the standard complex normal distribution;
- PU’s estimated channel \( a_j \sim \mathcal{CN}(0, I/\sqrt{\mathbb{N}}); \)
- noise variance \( \eta_i = 1 \) for all users;
- the transmit power \( P = 1; \)
- the upper bounds of IT \( \eta = d + n, \) where \( n \sim \mathcal{U}(0, 1) \) and \( d \sim \mathcal{U}(0, 1). \)

Besides, we randomly choose the same initial point, and set the same stopping criterion as

\[
\frac{|F(\mathbf{w}^k) - F(\mathbf{w}^{k-1})|}{F(\mathbf{w}^k)} \leq 10^{-4}, \quad k \geq 5.
\]

We repeat the tests 100 times and take the average.

**Figure 1:** The worst SUs’ SNR and computation time scale with \( N \) while \( M = 50 \) and \( J = 3. \)

**Figure 2:** The worst SUs’ SNR and computation time scale with \( M \) while \( N = 30 \) and \( J = 3. \)