Fast First-Order Methods for the Massive Robust Multicast Beamforming Problem with Interference Temperature Constraints

香港中文大學 The Chinese University of Hong Kong

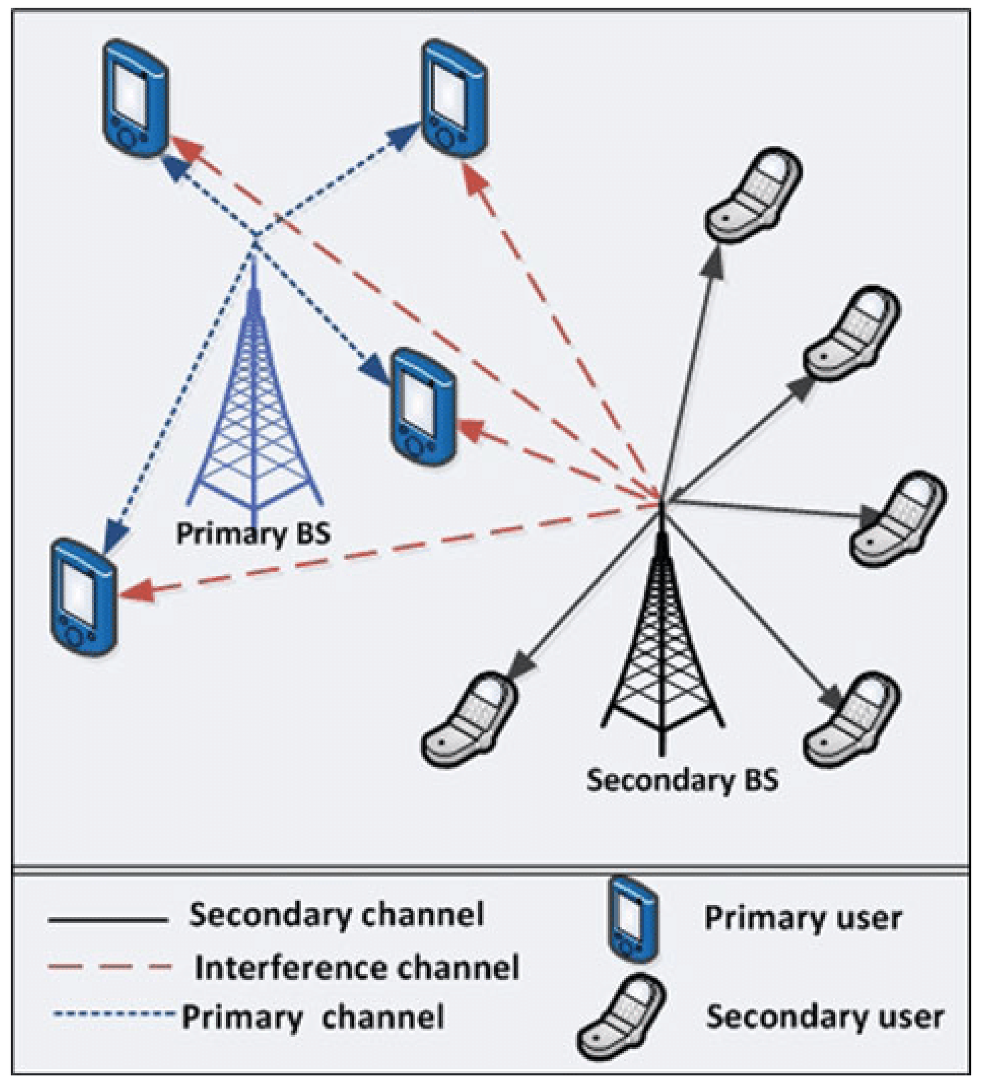
{ Huikang Liu, Peng Wang, Anthony Man-Cho So }   Dept. of System Erg. & Erg. Mgt., CUHK, Hong Kong

Introduction

We consider the following **robust beamforming problem with interference temperature** constraints in a cognitive radio network:

$$\max_{\boldsymbol{w}\in\mathbb{C}^N} \min_{i=1,\dots,M} |\boldsymbol{h}_i^H \boldsymbol{w}|^2/\sigma_i^2$$
$$\text{s.t.} \max_{\|\boldsymbol{\delta}_j\|\le d_j} |\boldsymbol{w}^H(\boldsymbol{a}_j+\boldsymbol{\delta}_j)|^2 \le \eta_j^2,\ j=1,\dots,J,$$
$$\|\boldsymbol{w}\|\le P,$$

- $\boldsymbol{w}\in\mathbb{C}^N$ is the beamforming vector;
- $\boldsymbol{h}_i$ is the channel between SBS and the ith SU;
- $\boldsymbol{a}_j$ is the estimated channel vector between the SBS and the jth PU;
- $\boldsymbol{\delta}_j$ is the estimation error bounded by d_j;
- P^2 is the average transmit power.



The interference temperature condition is equivalent to the following deterministic version:

$$d_j\|\boldsymbol{w}\| + \|\boldsymbol{a}_j^H\boldsymbol{w}\| \le \eta_j,\ \text{for } j=1,\dots,J.$$

Then, convert this problem into the real domain:

$$\min_{\boldsymbol{w}\in\mathbb{R}^{2N}} \max_{i=1,\dots,M} \boldsymbol{w}^T\boldsymbol{G}_i\boldsymbol{w}$$
$$\text{s.t. } d_j\|\boldsymbol{w}\| + \|\boldsymbol{a}_j^T\boldsymbol{w}\| \le \eta_j,\ \text{for } j=1,\dots,J,$$
$$\|\boldsymbol{w}\|\le P.$$

LPA-SD: Linear Programming-Assisted Subgradient Decent

We define

- $f_i(\boldsymbol{w}) = \boldsymbol{w}^T\boldsymbol{G}_i\boldsymbol{w},\ i=1,\dots,M$, and $F(\boldsymbol{w}) = \max_{i=1,\dots,M} f_i(\boldsymbol{w})$;
- δ-active objective functions set: $I_\delta(\boldsymbol{w}) := \{i\in\{1,\dots,M\} : f_i(\boldsymbol{w}) \ge F(\boldsymbol{w})-\delta\}$;
- ϵ-active constraints set: $I_\epsilon(\boldsymbol{w}) := \{j\in\{1,\dots,J+1\} : g_j(\boldsymbol{w}) \ge (1-\epsilon)\eta_j\}$;
- ϵ-tangent space at $\boldsymbol{w}$: $\mathcal{T}_\epsilon(\boldsymbol{w}) = \{\boldsymbol{x}\in\mathbb{R}^{2N} : \boldsymbol{x}^T\nabla g_j(\boldsymbol{w}) = 0, j\in I_\epsilon(\boldsymbol{w})\}$;
- projected gradient onto $\mathcal{T}_\epsilon(\boldsymbol{w})$: $\boldsymbol{p}_i(\boldsymbol{w}) = (\boldsymbol{I}-\boldsymbol{U}\boldsymbol{U}^T)\nabla f_i(\boldsymbol{w})$.

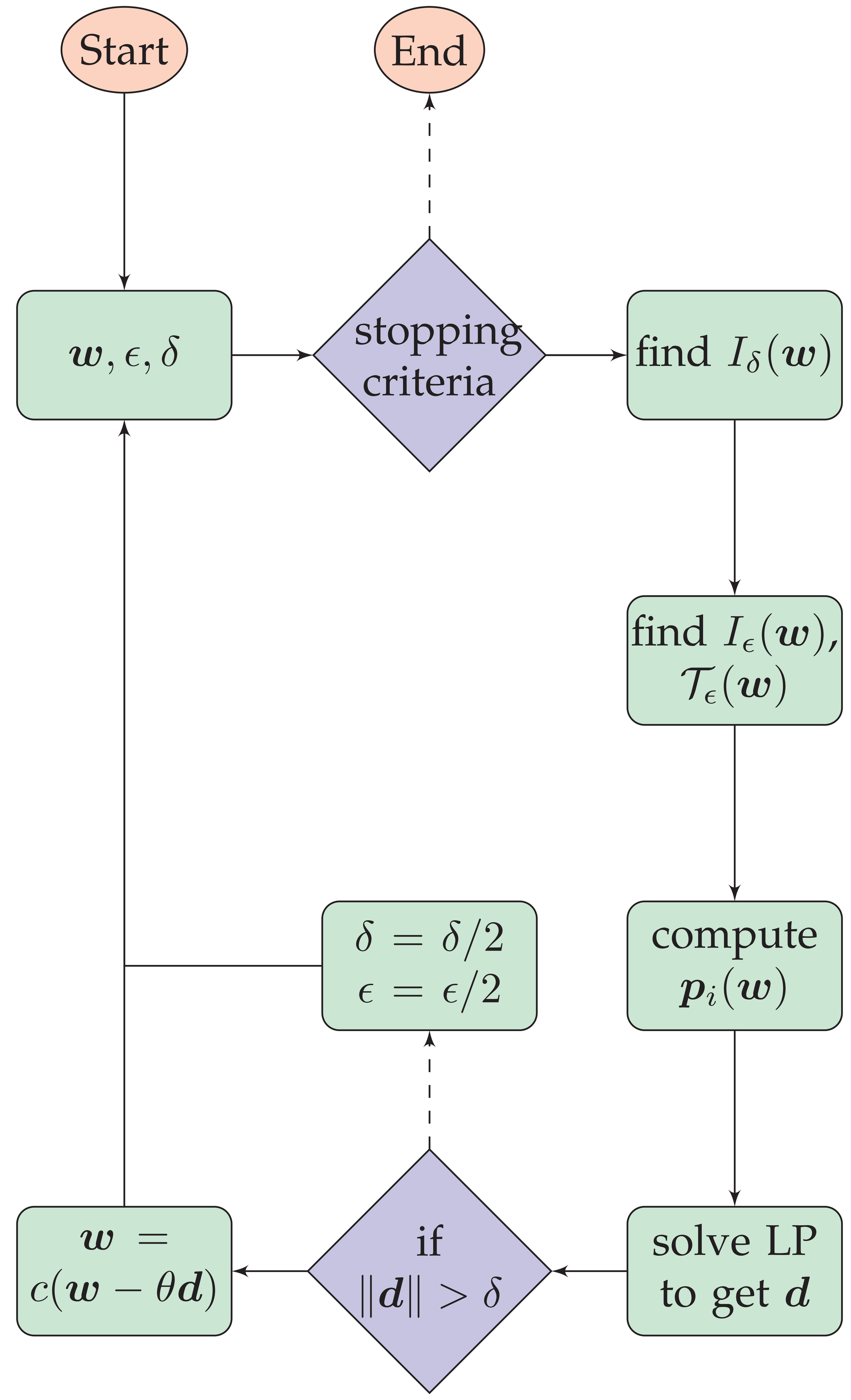
Then solve the following LP

$$\max_{t,\lambda}\ t$$
$$\text{s.t. } \boldsymbol{p}_i(\boldsymbol{w})^T \sum_{i\in I_\delta(\boldsymbol{w})} \lambda_i \boldsymbol{p}_i(\boldsymbol{w}) \ge t,$$
$$i\in I_\delta(\boldsymbol{w}),\ \lambda\in\Delta_{|I_\delta(\boldsymbol{w})|}.$$

Find the Armijo-type stepsize,

$$\bar\theta = \max_{l\ge 0}\{\theta^l : F(c(\boldsymbol{w}-\theta^l\boldsymbol{d})) \le F(\boldsymbol{w}) - \gamma\theta^l t^*\},$$

where $0<\theta<1$ and $0<\gamma\le 0.5$. Here, c is a scaling parameter to ensure the feasibility of $\tilde{\boldsymbol{w}} := \boldsymbol{w}-\theta^l\boldsymbol{d}$.



ADMM

Firstly, introducing the auxiliary and slackness variables to split the constraints

$$\min_{\boldsymbol{w},t,x,y,z,u,v}\ t$$
$$\text{s.t. } \|x_i\| - u_i = \sqrt{-t},\ i=1,\dots,M,$$
$$d_j\|y\| + \|z_j\| + v_j = \eta_j,\ j=1,\dots,J,$$
$$x_i = \boldsymbol{H}_i^T\boldsymbol{w},\ i=1,\dots,M,$$
$$y=\boldsymbol{w},\ z_j = \boldsymbol{A}_j^T\boldsymbol{w},\ j=1,\dots,J,$$
$$\|y\|\le P,\ u\ge 0,\ v\ge 0.$$

Write down the augmented Lagrangian function and apply the standard **multi-block ADMM**.

Numerical Result

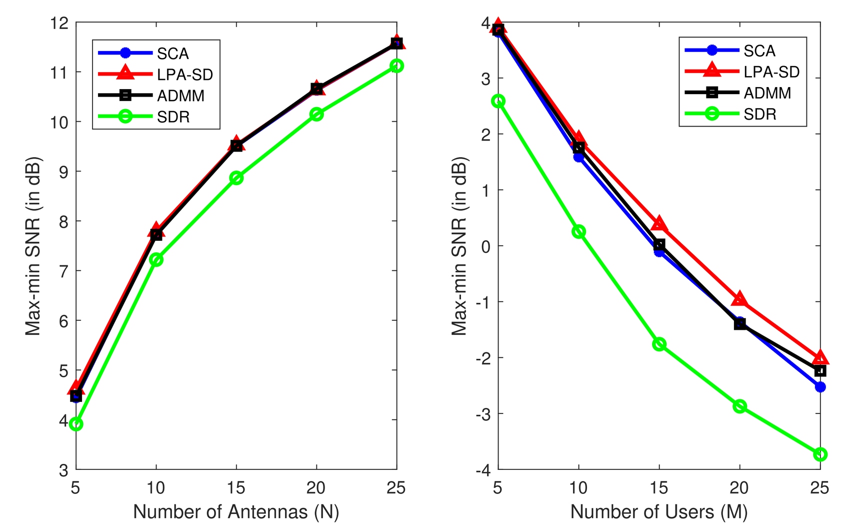


Figure 3: Small-size robust MIMO: (1) $M=5$, $J=3$; (2) $N=5$, $J=3$.

Numerical Result

We generate the data as follows:

- $\boldsymbol{h}_i \sim \mathcal{CN}(0,\boldsymbol{I})$ follows the standard complex normal distribution;
- PU's estimated channel $\boldsymbol{a}_j \sim \mathcal{CN}(0,\boldsymbol{I}/\sqrt{N})$;
- noise variance $\sigma_i = 1$ for all users;
- the transmit power $P=1$;
- the upper bounds of IT $\eta = d+n$, where $n\sim U(0,1)$ and $d\sim U(0,1)$.

Besides, we randomly choose the same initial point, and set the same stopping criterion as

$$|F(\boldsymbol{w}^k) - F(\boldsymbol{w}^{k-5})|/|F(\boldsymbol{w}^k)| \le 10^{-4},\ k\ge 5.$$

We repeat the tests **100** times and take the average.

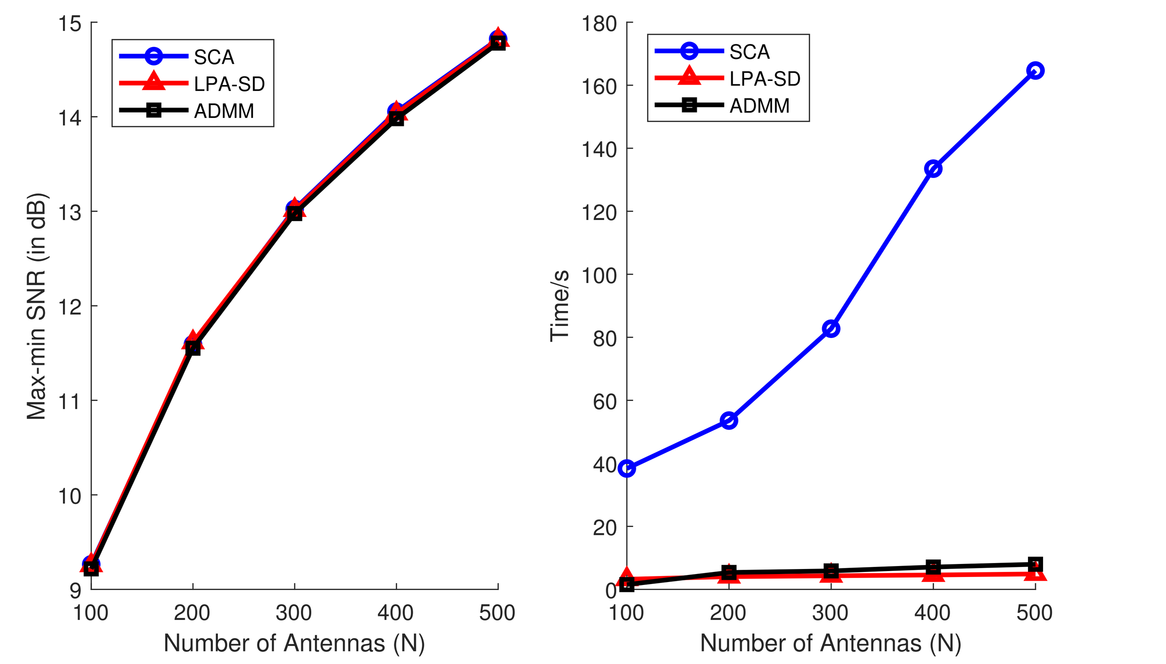


Figure 1: The worst SUs' SNR and computation time scale with N while $M=50$ and $J=3$.

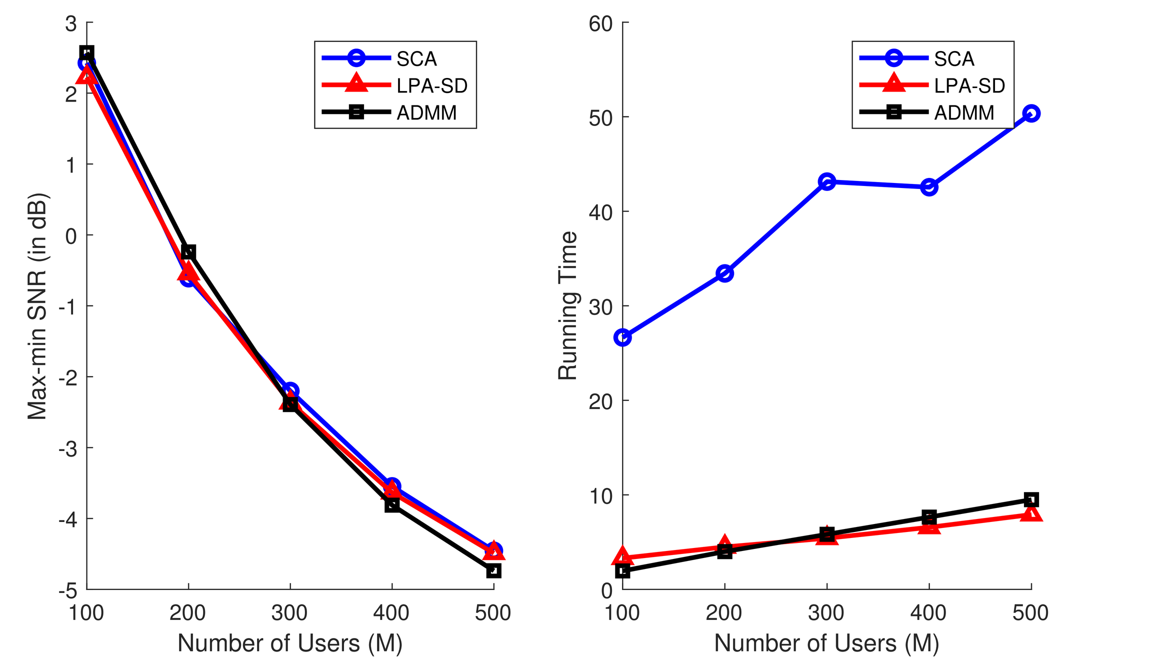


Figure 2: The worst SUs' SNR and computation time scale with M while $N=30$ and $J=3$